

periodicity of about 1300 Å is present (Fig. 8*a*). The 1300 Å unit is shown enlarged in Fig. 8*b*). This super-block itself consists of an irregular sequence of smaller units giving rise to lattice spacings of 10, 12.6, 15.1 and 17.6 Å corresponding to the 4*H*, 15*R*, 6*H* and 21*R* polytypes respectively. Another example shown in Fig. 9, having a 1242 Å periodicity (corresponding to 494 close-packed layers) is found to consist of 37 spacings of 6*H*, 32 of 15*R*, 10 of 21*R* and 8 of 4*H*. The frequency of occurrence of the polytypes constituting the super-blocks is such that 6*H* > 15*R* > 21*R* > 4*H*. Within such long-period supercells it is easy of course to introduce stacking faults or even to break down the periodicity over a limited distance. The presence of such faults will cause a supplementary streaking on the diffraction pattern. If more faults are present the streaking will become more pronounced and this will affect the intensities especially for X-ray diffraction where an average is taken over a larger area. Such a fault is indicated in Fig. 8*a*). If too many faults occur, the periodicity is lost and we have a completely long-range disordered crystal. During the present investigation using the lattice-imaging technique, crystals were found to be completely disordered over ranges of more than 5000 Å, *i.e.* no smaller repeat unit could be detected.

3. Conclusion

X-ray diffraction studies of single crystals of SiC have shown that the presence of diffuse streaks along $h0.l$ rows cannot necessarily be attributed to *complete* disorder. Careful analysis sometimes reveals discrete spots, suggesting the presence of long-period polytypes. When the periodicity becomes too large con-

ventional X-ray diffraction becomes very difficult and therefore the lattice-imaging technique by electron microscopy has been applied to determine the stacking of such extremely large polytypes. The different polytypes having periodicities of more than 1000 Å are built up of units of the 6*H*, 15*R*, 4*H* and 21*R* polytypes of SiC.

We are very grateful to Professor T. R. Anantharaman and Dr Suryanarayana for providing the electron microscope facility. One of us (MD) is grateful to CSIR for financial help during this period.

References

- ALLPRESS, J. G. & SANDERS, J. V. (1973). *J. Appl. Cryst.* **6**, 165–190.
 ANDERSON, J. S. & HUTCHISON, J. L. (1975). *Contemp. Phys.* **16**, 443–467.
 COWLEY, J. M. & IJIMA, S. (1972). *Z. Naturforsch.* **27a**, 445–451.
 RAM, U. S., DUBEY, M. & SINGH, G. (1973). *Z. Kristallogr.* **137**, 341–351.
 RAM, U. S., DUBEY, M. & SINGH, G. (1974). *Phys. Stat. Sol. (a)*, **22**, K125–K128.
 RAM, U. S. & SINGH, G. (1976). *J. Cryst. Growth*, **32**. In the press.
 SATO, H. & SHINOZAKI, S. (1975). *Mater. Res. Bull.* **10**, 257–260.
 VAN LANDUYT, J., AMELINCKX, S., KOHN, J. A. & ECKART, D. W. (1974). *J. Solid State Chem.* **9**, 103–119.
 VERMA, A. R. & KRISHNA, P. (1966). *Polymorphism and polytypism in Crystals*. New York: John Wiley.
 YESSIK, M., SHINOZAKI, S. & SATO, H. (1975). *Acta Cryst.* **A31**, 764–768.

Acta Cryst. (1977). **A33**, 279–287

Spin Point Groups

BY D. B. LITVIN

Department of Physics, University of British Columbia, Vancouver, BC, V6T 1W5, Canada

(Received 23 September 1975; accepted 21 April 1976)

The 598 classes of nontrivial spin point groups are derived and tabulated. The relationship between classes of nontrivial spin point groups and the 32 classes of trivial magnetic point groups and the 58 classes of nontrivial magnetic point groups is also given.

1. Introduction

The theory of *spin groups*, generalized magnetic groups defined to describe the symmetry of spin arrangements in crystals, has been given by Litvin & Opechowski

(1974). The theory of spin groups has been applied in the analysis of neutron diffraction data to determine spin arrangements in crystals. The so-called *nontrivial spin translation groups* and the possible magnetic reflexions of unpolarized neutrons elastically scattered

from a single crystal whose spin arrangement is invariant under a nontrivial spin translation group have been tabulated (Litvin, 1973). The symmetry of the spin density Patterson function due to the spin group symmetry of a spin arrangement has also been derived (Litvin, 1974).

In this paper we derive and tabulate the so called *nontrivial spin point groups*. We briefly review, in § 2, the theory of spin groups. In § 3 we review a method of deriving and then tabulate the 598 classes of nontrivial spin point groups. We also discuss the relationship between the classes of nontrivial spin point groups and the 32 classes of trivial magnetic point groups and the 58 classes of nontrivial magnetic point groups.

2. Spin groups

A group of elements $[R_1||R_2|v]$ is called a spin group if it is the symmetry group of some spin arrangement. In an element $[R_1||R_2|v]$ of a spin group, R_1 and R_2 are proper or improper rotation matrices, and v is a column matrix. When applying an element of a spin group to a spin arrangement, the rotation R_1 to the left of the double verticle bar is defined to act only in *spin space*, on the components of the spins, while the rotation and translation $(R_2|v)$ to the right is defined to act only in *physical space*, on the coordinates of the atoms. Applying a spin group element $[R_1||R_2|v]$ to a spin arrangement $S(\mathbf{r})$ therefore means replacing $S(\mathbf{r})$ by the spin arrangement denoted by $[R_1||R_2|v]S(\mathbf{r})$ and defined by (Litvin & Opechowski, 1974)*

$$[R_1||R_2|v]S^i(\mathbf{r}) = \sum_{j=1}^3 (R_1)_{ij} S^j[(R_2|v)^{-1}\mathbf{r}], \quad (1)$$

where $i=1, 2, 3$. A spin arrangement is said to be invariant under a spin element $[R_1||R_2|v]$ if

$$[R_1||R_2|v]S(\mathbf{r}) = S(\mathbf{r})$$

and the group of all such elements is called the *spin symmetry group* of the spin arrangement $S(\mathbf{r})$.

A spin group has been shown to be a direct product of a so-called *spin-only group* consisting of elements of the form $[R||E|0]$ and a second group called a *nontrivial spin group* (Litvin & Opechowski, 1974). A nontrivial spin group contains elements of the form $[R_1||R_2|v]$ but no elements of the form $[R||E|0]$ with $R \neq E$. A *nontrivial spin point group* is a nontrivial spin group consisting of elements of the form $[R_1||R_2|0]$.

Since only nontrivial spin point groups will be considered in the remainder of this paper we will drop the adjective 'nontrivial'. Thus, in the remainder of this paper 'spin point group' will mean 'nontrivial spin point group'. We will also denote an element of a spin

point group by $[R_1||R_2]$ instead of the longer notation $[R_1||R_2|0]$.

3. Spin point groups

A spin point group, denoted by \mathbf{R}_s is said to belong to the *family* of \mathbf{B} and \mathbf{R} if the left-hand components of the elements $[R_1||R_2]$ of \mathbf{R}_s constitute the group \mathbf{B} , and the right-hand components the group \mathbf{R} . Both \mathbf{B} and \mathbf{R} are point groups belonging to one of the 32 classes of point groups. A spin point group contains a normal subgroup consisting of all elements of \mathbf{R}_s of the form $[E||R]$. The right-hand components of elements of this normal subgroup constitute a normal subgroup \mathbf{r} of \mathbf{R} , and is such that $\mathbf{R}/\mathbf{r} \approx \mathbf{B}$, i.e. the factor group \mathbf{R}/\mathbf{r} is isomorphic with \mathbf{B} .

To derive all spin point groups one proceeds as follows (Litvin & Opechowski, 1974). For a specific group \mathbf{R} one finds all normal subgroups \mathbf{r} of \mathbf{R} and then those groups \mathbf{B} which are isomorphic with \mathbf{R}/\mathbf{r} . Each pair of groups \mathbf{B} and \mathbf{r} , and each isomorphism between \mathbf{B} and \mathbf{R}/\mathbf{r} determines a spin point group belonging to the family of \mathbf{B} and \mathbf{R} . One writes \mathbf{R} as a coset decomposition with respect to \mathbf{r}

$$\mathbf{R} = \mathbf{r} + R_2\mathbf{r} + \dots + R_n\mathbf{r}$$

and \mathbf{B} as

$$\mathbf{B} = E + B_2 + \dots + B_n,$$

where the cosets $R_i\mathbf{r}$ and elements B_i are mapped on each other by the isomorphism $\mathbf{R}/\mathbf{r} \approx \mathbf{B}$. To construct the spin point group one then pairs the element B_i of \mathbf{B} with each element of the i th coset of \mathbf{R} . The spin point group so derived is

$$\mathbf{R}_s = [E||\mathbf{r}] + [B_2||R_2][E||\mathbf{r}] + \dots + [B_n||R_n][E||\mathbf{r}],$$

where $[E||\mathbf{r}]$ denotes the normal subgroup of \mathbf{R}_s consisting of all elements of \mathbf{R}_s of the form $[E||R]$. By taking in turn a point group \mathbf{R} from each of the 32 classes of point groups, one finds all spin point groups.

For the purpose of classifying spin point groups we denote by \mathcal{H} the linear group $GL(3)$ in spin space, and by \mathcal{A} the affine group $GIL(3)$ in physical space. Spin point groups are subgroups of $\mathcal{H} \times \mathcal{A}$, and are classified into classes of conjugate subgroups of $\mathcal{H} \times \mathcal{A}$ (Litvin & Opechowski, 1974). That is, two spin point groups are said to belong to the same 'class of spin point groups' if they are conjugate subgroups of $\mathcal{H} \times \mathcal{A}$. In Table 1 we list the 598 classes of spin point groups.*

In the first three columns of Table 1 trios of point groups \mathbf{R} , \mathbf{r} , and \mathbf{B} are listed where \mathbf{R}/\mathbf{r} is isomorphic with \mathbf{B} . These point groups are given in the 'short' international notation except in the case of two cubic point groups which are given in 'full' international

* It should be pointed out that definition (1) is somewhat different to that introduced by Litvin (1973). This modification does not require any changes in Tables 1 and 2 of Litvin (1973) except replacing primes by horizontal bars, i.e. Q' is replaced by \bar{Q} .

* The number of classes of spin point groups quoted by Litvin & Opechowski (1974) and based on the work of Litvin (1971) is not correct.

Table 1. *Classes of spin point groups*

Point groups **R**, **r**, and **B** are listed in the first three columns. In column four, a representative spin point group from each of the classes of spin point groups determined by isomorphisms between **R/r** and **B** is listed. In column five, the 32 classes of trivial magnetic point groups and 58 classes of nontrivial magnetic point groups are listed alongside the classes of spin point groups to which they belong.

	R	r	B			R	r	B		
1.	1	1	1	1_1	1	51.	1	222	$2x_2^2y_2^2z_2$	222
2.	$\bar{1}$	$\bar{1}$	1	$1_1\bar{1}$	$\bar{1}$	52.		mm2	$mx_2^2my_2^2z_2$	$2'2'2$
3.		1	2	$2_1\bar{1}$		53.		2/m	$2_2\bar{1}2_2$	
4.			m	$m_1\bar{1}$		54.	mmm	mmm	$1_1 1_1 1_1$	
5.			$\bar{1}$	$\bar{1}_1$	$\bar{1}'$	55.		2/m	$1_1 2_1 2_1$	
6.	2	2	1	1_2		56.		m	$1_1 m m$	
7.		1	2	2_2	2	57.		$\bar{1}$	$1_1 \bar{1} \bar{1}$	
8.			m	m_2	$2'$	58.		mm2	$1_1 1_1 2_1$	
9.			$\bar{1}$	$\bar{1}_2$		59.		m	$1_1 1_1 m$	
10.	m	m	1	1_m		60.		$\bar{1}$	$1_1 1_1 \bar{1}$	
11.		1	2	2_m	m	61.		222	$2_2 2_2 2_2$	
12.			m	m_m	m'	62.		m	$m_m m_m m_m$	
13.			$\bar{1}$	$\bar{1}_m$		63.		$\bar{1}$	$\bar{1}_m \bar{1}_m \bar{1}_m$	
14.	2/m	2/m	1	$1_2/1_m$		64.		2	$2x_m 2y_m 2z_m$	
15.		2	2	$1_2/2_m$		65.		mm2	$my_m my_m mx_m$	
16.			m	$1_2/m_m$		66.			$my_m my_m 2z_m$	
17.			$\bar{1}$	$1_2/\bar{1}_m$		67.			$2x_m 2y_m my_m$	
18.		m	2	$2_2/1_m$		68.		2/m	$m_m m_m \bar{1}$	
19.			m	$m_2/1_m$		69.			$m_m m_m 2_2$	
20.			$\bar{1}$	$\bar{1}_2/1_m$		70.			$2_2 2_2 \bar{1}$	
21.		$\bar{1}$	2	$2_2/2_m$	2/m	71.			$2_2 2_2 m_m$	
22.			m	m_2/m_m	$2'/m'$	72.			$\bar{1}_m \bar{1}_m 2_2$	
23.			$\bar{1}$	$\bar{1}_2/\bar{1}_m$		73.			$\bar{1}_m \bar{1}_m m_m$	
24.		1	222	$2x_2/2x_m$		74.		m	$2x_m 2y_m 1_m$	
25.			mm2	$2x_2/mx_m$		75.		mm2	$mx_m my_m 1_m$	
26.				mx_2/my_m		76.			$my_m 2x_m 1_m$	
27.				$mx_2/2z_m$		77.		2/m	$m_m \bar{1} 1_m$	
28.			2/m	$2_2/m_m$	2/m'	78.			$m_m 2_2 1_m$	
29.				$2_2/\bar{1}_m$		79.			$\bar{1}_m 2_2 1_m$	
30.				$\bar{1}_2/m_m$		80.		$\bar{1}$	$2x_m 2y_m 2z_m$	mmm
31.				$\bar{1}_2/2_m$		81.		mm2	$mx_m my_m 2z_m$	$m'm'm$
32.				$m_2/\bar{1}_m$		82.		2/m	$2_2 \bar{1}_m m_m$	
33.				$m_2/2_m$	$2'/m'$	83.		1	$mx_m my_m mx_m$	$m'm'm'$
34.	mm2	mm2	1	$1_m 1_m 1_2$		84.			$mx_m 2y_m 2z_m$	$m'mm$
35.		2	2	$2_2 m 1_2$		85.			$mx_m 2x_m 2z_m$	
36.			m	$m_m m 1_2$		86.			$2z_m mx_m mx_m$	
37.			$\bar{1}$	$\bar{1}_m \bar{1}_m 1_2$		87.			$2z_m \bar{1}_m mx_m$	
38.		m	2	$1_2 m 2_2$		88.			$2z_m \bar{1}_m 2x_m$	
39.			m	$1_m m m_2$		89.			$\bar{1}_m mx_m mx_m$	
40.			$\bar{1}$	$1_m \bar{1}_m 2_2$		90.	4	4	1_4	
41.		1	222	$2x_m 2y_m 2z_2$	mm2	91.		2	2_4	
42.			mm2	$mx_m my_m 2z_2$	$m'm'2$	92.		m	m_4	
43.				$mx_m 2z_m my_2$	$m'm2'$	93.		$\bar{1}$	$\bar{1}_4$	
44.			2/m	$m_m \bar{1}_m 2_2$		94.		1	4_4	4
45.				$m_m 2_2 \bar{1}_m$		95.		$\bar{4}$	$\bar{4}_4$	$4'$
46.				$\bar{1}_m 2_2 m_m$		96.	$\bar{4}$	$\bar{4}$	1_4	
47.	222	222	1	$1_2 1_2 1_2$		97.		2	2_4	
48.		2	2	$1_2 2_2 2_2$		98.		m	m_4	
49.			m	$1_2 m_2 m_2$		99.		$\bar{1}$	$\bar{1}_4$	
50.			$\bar{1}$	$1_2 \bar{1}_2 \bar{1}_2$		100.		1	4_4	$\bar{4}$

Table 1 (cont.)

R	r	B			R	r	B		
101.		$\bar{4}$	$\bar{4}_x$	$\bar{4}'$	151.		m	$m_4^1 m^m m$	
102.	4/m	4/m	1	$1_4/1_m$	152.		$\bar{1}$	$\bar{1}_4^1 \bar{1}_m^1$	
103.		2/m	2	$2_4/1_m$	153.	2	222	$2x_4^2 y_m^2 z_m^2$	
104.		m	$m_4/1_m$		154.		mm2	$2x_4^2 m^m y_m$	
105.		$\bar{1}$	$\bar{1}_4/1_m$		155.			$m x_4^2 z_m^2 m^m y_m$	
106.	$\bar{4}$	2	$2_4/2_m$		156.	2/m		$2_4 \bar{1}_m^m m$	
107.		m	m_4/m^m		157.			$\bar{1}_4^2 m^m m$	
108.		$\bar{1}$	$\bar{1}_4/\bar{1}_m$		158.			$m_4^2 m^1 m$	
109.	4	2	$1_4/2_m$		159.	1	422	$4x_4^2 z_m^2 x^m y_m$	4mm
110.		m	$1_4/m^m$		160.		4mm	$4x_4^2 m^m m^m x y_m$	4m'm'
111.		$\bar{1}$	$1_4/\bar{1}_m$		161.		$\bar{4}2m$	$\bar{4}x_4^2 m^m x^m y_m$	4'm'm'
112.	$\bar{1}$	4	$4_4/2_m$	4/m	162.	$\bar{4}2m$	$\bar{4}2m$	1	$1_4^2 1_m^1$
113.		$\bar{4}$	$\bar{4}_4/2_m$	4'/m	163.		$\bar{4}$	2	$1_4^2 2_m^2$
114.		m	$4_4/1_m$		164.		m	$1_4^m 2_m^m$	
115.		$\bar{4}$	$\bar{4}_4/1_m$		165.		$\bar{1}$	$1_4^2 \bar{1}_m^1$	
116.	2	222	$2x_4^2/2x_m^2$		166.		mm2	2	$2x_4^2 2_m^1$
117.		mm2	$2x_4^2/m^m x_m$		167.		m	$m_4^2 m^1 m$	
118.			$m x_4^2/2z_m$		168.		$\bar{1}$	$\bar{1}_4^2 1_m^1$	
119.			$m x_4^2/m^m y_m$		169.	222	2	$2_4^2 2_m^2$	
120.		2/m	$2_4/\bar{1}_m$		170.		m	$m_4^2 2_m^m$	
121.			$2_4/m^m$		171.		$\bar{1}$	$\bar{1}_4^2 2_m^1$	
122.			$\bar{1}_4/2_m$		172.	2	222	$2x_4^2 y^2 z^2$	
123.			$\bar{1}_4/m^m$		173.		mm2	$2z_4^2 x^2 m^m y_m$	
124.			$m_4/2_m$		174.			$m x_4^2 z^2 m^m y_m$	
125.			$m_4/\bar{1}_m$		175.			$m x_4^2 m^m y^2 z^2$	
126.	1	4/m	$4_4/m^m$	4/m'	176.		2/m	$2_4^2 2_m^m$	
127.			$4_4/\bar{1}_m$		177.			$2x_4^2 m^1 m$	
128.			$\bar{4}_4/m^m$	4'/m'	178.			$\bar{1}_4^2 2_m^m$	
129.			$\bar{4}_4/\bar{1}_m$		179.			$\bar{1}_4^m 2_m^2$	
130.	422	422	1	$1_4^1 2^1 2$	180.			$m_4^2 2_m^1$	
131.		4	2	$1_4^2 2^2 2$	181.			$m_4^2 2_m^2$	
132.		m	$1_4^m 2_m^m 2$		182.	1	422	$4x_4^2 x^2 z^2 x y_m$	$\bar{4}2m$
133.		$\bar{1}$	$1_4^2 2^1 2$		183.		4mm	$4x_4^2 m^m x^2 m^m x y_m$	$\bar{4}2'm'$
134.		222	$2_4^1 2^2 2$		184.		$\bar{4}2m$	$\bar{4}x_4^2 x^2 m^m x y_m$	$\bar{4}'2m'$
135.		m	$m_4^1 2_m^m 2$		185.			$\bar{4}x_4^2 m^m x^2 z^2 x y_m$	$\bar{4}'2'm'$
136.		$\bar{1}$	$\bar{1}_4^1 2^1 2$		186.	4/mmm	4/mmm	1	$1_4/1_m^1 1_m^1$
137.		2	$2x_4^2 y^2 z^2$		187.		$\bar{4}2m$	2	$2_4/2_m^2 1_m^1$
138.		mm2	$2x_4^2 m^m x^2 m^m y^2$		188.		m	$m_4/m^m m^m 1_m$	
139.			$m x_4^2 z^2 m^m y^2$		189.		$\bar{1}$	$\bar{1}_4/\bar{1}_m^1 1_m^1$	
140.		2/m	$2_4^2 2_m^2$		190.		4mm	2	$1_4/2_m^2 1_m^1$
141.			$\bar{1}_4^2 2_m^2$		191.		m	$1_4/m^m 1_m^1$	
142.			$m_4^2 2_m^2$		192.		$\bar{1}$	$1_4/\bar{1}_m^1 1_m^1$	
143.	1	422	$4x_4^2 x^2 z^2 x y^2$	422	193.		mmm	2	$2_4/1_m^1 1_m^2$
144.		4mm	$4x_4^2 m^m x^2 m^m x y_m$	42'2'	194.		m	$m_4/1_m^1 m^m m$	
145.		$\bar{4}2m$	$\bar{4}x_4^2 x^2 m^m x y^2$	4'22'	195.		$\bar{1}$	$\bar{1}_4/1_m^1 1_m^1$	
146.	4mm	4mm	1	$1_4/1_m^1 1_m^1$	196.	4/m	2	$1_4/1_m^1 2_m^2$	
147.		4	2	$1_4^2 2_m^2$	197.		m	$1_4/1_m^m m^m m$	
148.		m	$1_4^m m^m m$		198.		$\bar{1}$	$1_4/1_m^1 \bar{1}_m^1$	
149.		$\bar{1}$	$1_4/\bar{1}_m^1 m$		199.	422	2	$1_4/2_m^2 2_m^2$	
150.		mm2	$2_4^1 2_m^2$		200.		m	$1_4/m^m m^m m$	

Table 1 (cont.)

R	r	B		R	r	B	
201.		$\bar{1}$	$1_4/\bar{1}_m\bar{1}_m$	251.		$m_{x_4}/m_{x_m}^2x_m^2y_m$	
202.	$\bar{4}$	222	$2z_4/2x_m^2x_m^2y_m$	252.		$2z_4/m_{x_m}^2x_m^2y_m$	
203.		$mm2$	$m_{x_4}/2x_m^2x_m^2y_m$	253.		$m_{x_4}/2x_m^2x_m^2y_m$	
204.			$m_{x_4}/m_{x_m}^2x_m^2y_m$	254.		$2z_4/m_{x_m}^2x_m^2y_m$	
205.	$2/m$		$2_4/2\bar{1}_m\bar{1}_m$	255.		$2z_4/m_{x_m}^2x_m^2y_m$	
206.			$\bar{1}_4/\bar{1}_m^2\bar{1}_m$	256.		$m_{x_4}/2x_m^2x_m^2\bar{1}_m$	
207.			$m_4/m^2\bar{1}_m$	257.		$m_{x_4}/m_{x_m}^2x_m^2z_m$	
208.	4	222	$1_4/2x_m^2y_m^2y_m$	258.		$m_{x_4}/\bar{1}_m^2x_m^2y_m$	
209.		$mm2$	$1_4/2x_m^2x_m^2x_m$	259.		$2z_4/2x_m^2x_m^2y_m$	
210.			$1_4/m_{x_m}^2x_m^2z_m$	260.		$2z_4/2x_m^2x_m^2\bar{1}_m$	
211.			$1_4/m_{x_m}^2y_m^2y_m$	261.		$2z_4/\bar{1}_m^2x_m^2y_m$	
212.	$2/m$		$1_4/2\bar{1}_m\bar{1}_m$	262.		$2z_4/m_{x_m}^2x_m^2z_m$	
213.			$1_4/2\bar{1}_m\bar{1}_m$	263.		$2z_4/\bar{1}_m^2x_m^2y_m$	
214.			$1_4/\bar{1}_m^2\bar{1}_m$	264.		$\bar{1}_4/2x_m^2x_m^2z_m$	
215.			$1_4/\bar{1}_m\bar{1}_m$	265.		$\bar{1}_4/m_{x_m}^2x_m^2z_m$	
216.			$1_4/m^2\bar{1}_m$	266.	1	$4z_4/m_{x_m}^2x_m^2xy_m$	$4/m^2m'$
217.			$1_4/m\bar{1}_m\bar{1}_m$	267.		$4z_4/\bar{1}_m^2x_m^2xy_m$	
218.	$2/m$	222	$2z_4/1_m^2x_m^2y_m$	268.		$4z_4/m_{x_m}^2x_m^2xy_m$	$4/m^2mm$
219.		$mm2$	$2z_4/1_m^2x_m^2y_m$	269.		$4z_4/\bar{1}_m^2x_m^2xy_m$	
220.			$m_{x_4}/1_m^2x_m^2y_m$	270.		$4z_4/m_{x_m}^2x_m^2xy_m$	$4'/m^2m'$
221.	$2/m$		$2_4/1_m\bar{1}_m$	271.		$4z_4/\bar{1}_m^2x_m^2xy_m$	
222.			$\bar{1}_4/1_m^2\bar{1}_m$	272.	3	1_3	
223.			$m_4/1_m^2\bar{1}_m$	273.	1	3_3	3
224.	$mm2$	222	$2z_4/2x_m^2x_m^2z_m$	274.	$\bar{3}$	1_3	
225.		$mm2$	$2z_4/m_{x_m}^2x_m^2z_m$	275.	3	2_3	
226.			$m_{x_4}/2x_m^2x_m^2z_m$	276.		m_3	
227.			$m_{x_4}/m_{x_m}^2x_m^2z_m$	277.		$\bar{1}_3$	$\bar{3}$
228.	$2/m$		$2_4/\bar{1}_m^2\bar{1}_m$	278.	$\bar{1}$	3_3	
229.			$2_4/m^2\bar{1}_m$	279.	1	6_3	
230.			$\bar{1}_4/2\bar{1}_m\bar{1}_m$	280.		$\bar{3}_3$	$\bar{3}'$
231.			$\bar{1}_4/m^2\bar{1}_m$	281.		6_3	
232.			$m_4/2\bar{1}_m\bar{1}_m$	282.	32	1_3^2	
233.			$m_4/\bar{1}_m^2\bar{1}_m$	283.	3	1_3^2	
234.	222	222	$2z_4/2x_m^2x_m^2y_m$	284.		1_3^2	
235.		$mm2$	$2z_4/m_{x_m}^2x_m^2y_m$	285.		$1_3\bar{1}_2$	
236.			$m_{x_4}/2x_m^2x_m^2y_m$	286.	1	$3z_3^2x_2$	32
237.			$m_{x_4}/m_{x_m}^2x_m^2y_m$	287.		$3z_3^2x_2$	$32'$
238.	$2/m$		$2_4/\bar{1}_m\bar{1}_m$	288.	$3m$	1_3^2	
239.			$2_4/m^2\bar{1}_m$	289.	3	1_3^2	
240.			$\bar{1}_4/2\bar{1}_m\bar{1}_m$	290.		1_3^2m	
241.			$\bar{1}_4/m^2\bar{1}_m$	291.		$1_3\bar{1}_m$	
242.			$m_4/2\bar{1}_m\bar{1}_m$	292.	1	$3z_3^2x_m$	$3m$
243.			$m_4/\bar{1}_m^2\bar{1}_m$	293.		$3z_3^2x_m$	$3m'$
244.	m	422	$4z_4/1_m^2x_m^2xy_m$	294.	$\bar{3}m$	1_3^2	
245.		$4mm$	$4z_4/m_{x_m}^2x_m^2xy_m$	295.		1_3^2m	
246.		$\bar{4}2m$	$4z_4/1_m^2x_m^2xy_m$	296.		1_3^2m	
247.	$\bar{1}$	422	$4z_4/2x_m^2x_m^2xy_m$	297.		$1_3\bar{1}_m$	
248.		$4mm$	$4z_4/m_{x_m}^2x_m^2xy_m$	298.	$3m$	2_3^2	
249.		$\bar{4}2m$	$4z_4/2x_m^2x_m^2xy_m$	299.		m_3^2	
250.	2	$mmmm$	$m_{x_4}/2x_m^2x_m^2xy_m$	300.		$\bar{1}_3^2$	

Table 1 (cont.)

R	r	B		R	r	B		
301.	32	2	$2_{\bar{3}}^2m$	351.	2	32	$3_26^2x_2^2xy_2$	
302.		m	$m\bar{3}m$	352.		3m	$3_26^m x_2^2 m xy_2$	
303.		$\bar{1}$	$\bar{1}\bar{3}^1m$	353.	1	622	$6_26^2x_2^2 2^2 1_2$	622
304.	3	222	$2_x 3^2 2_m$	354.		$\bar{3}m$	$\bar{3}_26^2x_2^2 m xy_2$	
305.		mm2	$2_x 3^m x_m$	355.		6mm	$6_26^m x_2^2 m 1_2$	62'2'
306.			$m_x 3^m y_m$	356.		$\bar{6}m2$	$\bar{6}_26^m x_2^2 2^2 1_2$	6'2'2
307.			$m_x 3^2 z_m$	357.	6/m	6/m	$1_6/1_m$	
308.	2/m		$2_3^m m$	358.		$\bar{3}$	$2_6/2_m$	
309.			$2_{\bar{3}}^1 m$	359.		m	m_6/m_m	
310.			$\bar{1}\bar{3}^m m$	360.		$\bar{1}$	$\bar{1}_6/\bar{1}_m$	
311.			$\bar{1}\bar{3}^2 m$	361.	$\bar{6}$	2	$2_6/1_m$	
312.			$m\bar{3}^1 m$	362.		m	$m_6/1_m$	
313.			$m\bar{3}^2 m$	363.		$\bar{1}$	$\bar{1}_6/1_m$	
314.	$\bar{1}$	32	$3_x 3^2 x_m$	364.	6	2	$1_6/2_m$	
315.		3m	$3_x 3^m x_m$	365.		m	$1_6/m_m$	
316.	1	622	$6_x 3^2 x_m$	366.		$\bar{1}$	$1_6/\bar{1}_m$	
317.		$\bar{3}m$	$\bar{3}_x 3^m x_m$	367.		2/m	$3_6/1_m$	
318.			$\bar{3}_x 3^2 x_m$	368.	3	222	$2_x 6/2_x m$	
319.		6mm	$6_x 3^m x_m$	369.		mm2	$2_x 6^m x_m$	
320.		6m2	$\bar{6}_x 3^m x_m$	370.			$m_x 6/2_x m$	
321.			$\bar{6}_x 3^2 x_m$	371.			$m_x 6^m y_m$	
322.	$\bar{6}$	$\bar{6}$	$1_{\bar{6}}$	372.		2/m	$2_6/1_m$	
323.		3	$2_{\bar{6}}$	373.			$2_6/m_m$	
324.		m	$m_{\bar{6}}$	374.			$\bar{1}_6/2_m$	
325.		$\bar{1}$	$\bar{1}_{\bar{6}}$	375.			$\bar{1}_6/m_m$	
326.		m	$3_{\bar{6}}$	376.			$m_6/2_m$	
327.	1	6	$6_{\bar{6}}$	377.			$m_6/\bar{1}_m$	
328.			$\bar{3}_{\bar{6}}$	378.	2	6	$3_6/2_m$	
329.			$\bar{6}_{\bar{6}}$	379.		$\bar{3}$	$3_6/\bar{1}_m$	
330.	6	6	1_6	380.		$\bar{6}$	$3_6/m_m$	
331.		3	2_6	381.		m	$6_6/1_m$	
332.		m	m_6	382.		$\bar{3}$	$\bar{3}_6/1_m$	
333.		$\bar{1}$	$\bar{1}_6$	383.		$\bar{6}$	$\bar{6}_6/1_m$	
334.	2	3	3_6	384.	$\bar{1}$	6	$6_6/2_m$	6/m
335.	1	6	6_6	385.		$\bar{3}$	$\bar{3}_6/\bar{1}_m$	
336.			$\bar{3}_6$	386.		$\bar{6}$	$\bar{6}_6/m_m$	6'/m'
337.			$\bar{6}_6$	387.	1	6/m	$6_6/m_m$	6'/m'
338.	622	622	$1_6^1 2^1 1_2$	388.			$6_6/\bar{1}_m$	
339.		6	$1_6^2 2^2 2_2$	389.			$\bar{3}_6/m_m$	
340.		m	$1_6^m 2^m 2_2$	390.			$\bar{3}_6/2_m$	
341.			$1_6^1 2^1 2_2$	391.			$\bar{6}_6/\bar{1}_m$	
342.		32	$2_6^1 2^2 2_2$	392.			$\bar{6}_6/2_m$	6'/m
343.		m	$m_6^1 2^m 2_2$	393.	6mm	6mm	$1_6^1 1_m 1_m$	
344.		$\bar{1}$	$\bar{1}_6^1 2^1 2_2$	394.		6	$1_6^2 2_m 1_m$	
345.		3	$2_x 6^2 y_2^2 z_2$	395.		m	$1_6^m m m$	
346.		mm2	$2_x 6^m x_2^m y_2$	396.		$\bar{1}$	$1_6^1 \bar{1}_m 1_m$	
347.			$m_x 6^2 z_2^m y_2$	397.		3m	$2_6^1 2_m 2_m$	
348.		2/m	$2_6^1 \bar{1}_2 m_2$	398.		m	$m_6^1 m m$	
349.			$\bar{1}_6^2 2^m m_2$	399.		$\bar{1}$	$\bar{1}_6^1 \bar{1}_m 1_m$	
350.			$m_6^2 2^1 2_2$	400.		3	$2_x 6^2 y_m^2 z_m$	

Table 1 (cont.)

R	r	B		R	r	B	
401.		mm2	$2z_6^{m_x m_y m_z}$	451.		m	$1_6/m^1_1 m^1_1$
402.			$m_x 6^2 z_6^{m_y m_z}$	452.		$\bar{1}$	$1_6/\bar{1}_1 m^1_1 m^1_1$
403.		2/m	$2_6^1 m^1 m^1$	453.	622	2	$1_6/2_6^2 m^1 m^1 m^1$
404.			$\bar{1}_6^2 m^1 m^1$	454.		m	$1_6/m^1 m^1 m^1$
405.			$m_6^2 \bar{1}_6^1$	455.		$\bar{1}$	$1_6/\bar{1}_1 m^1 m^1$
406.	2	32	$3z_6^2 z_6^{m_x m_y m_z}$	456.	$\bar{3}$	222	$2z_6/2_6^2 z_6^{m_x m_y m_z}$
407.		3m	$3z_6^{m_x m_y m_z}$	457.		mm2	$2z_6/2_6^2 z_6^{m_x m_y m_z}$
408.	1	622	$6z_6^2 z_6^{m_x m_z}$	458.			$m_x 6/m_x m^2 z_6^{m_y m_z}$
409.		$\bar{3}m$	$\bar{3}z_6^2 z_6^{m_x m_y m_z}$	459.		2/m	$2_6/2_6^2 m^1 m^1 m^1$
410.		6mm	$6z_6^{m_x m_z}$	460.			$\bar{1}_6/\bar{1}_1 m^1 m^1 m^1$
411.		$\bar{6}m2$	$\bar{6}z_6^{m_x m_z}$	461.			$m_6/m^1 m^2 \bar{1}_6^1$
412.	$\bar{6}m2$	$\bar{6}m2$	$1_6^1 m^1 m^1$	462.	$\bar{6}$	222	$2z_6/m^1 m^2 z_6^{m_y m_z}$
413.		$\bar{6}$	$1_6^2 m^1 m^1$	463.		mm2	$2z_6/1_6^1 m_x m_y m_z$
414.		m	$1_6^m m^1 m^1$	464.			$m_x 6/1_6^2 z_6^{m_y m_z}$
415.		$\bar{1}$	$1_6^1 m^1 m^1$	465.		2/m	$2_6/1_6^1 m^1 m^1 m^1$
416.		3m	$2_6^1 m^1 m^1$	466.			$\bar{1}_6/1_6^1 m^1 m^1 m^1$
417.		m	$m_6^1 m^1 m^1$	467.			$m_6/1_6^1 m^1 m^1 m^1$
418.		$\bar{1}$	$\bar{1}_6^1 m^1 m^1$	468.	6	222	$1_6/2_6^2 z_6^{m_x m_y m_z}$
419.		32	$2_6^2 m^1 m^1$	469.		mm2	$1_6/2_6^2 z_6^{m_x m_y m_z}$
420.		m	$m_6^m m^1 m^1$	470.			$1_6/m_x m_y m_z$
421.		$\bar{1}$	$\bar{1}_6^1 m^1 m^1$	471.			$1_6/m_x m^2 z_6^{m_y m_z}$
422.		3	$2z_6^2 z_6^{m_y m_z}$	472.		2/m	$1_6/2_6^2 m^1 m^1 m^1$
423.		mm2	$2z_6^{m_x m_y m_z}$	473.			$1_6/2_6^2 m^1 m^1 m^1$
424.			$m_x 6^{m_y m_z}$	474.			$1_6/\bar{1}_1 m^1 m^1 m^1$
425.			$m_x 6^2 z_6^{m_y m_z}$	475.			$1_6/\bar{1}_1 m^2 m^1 m^1$
426.		2/m	$2_6^m m^1 m^1$	476.			$1_6/m^1 m^1 m^1$
427.			$2_6^1 m^1 m^1$	477.			$1_6/m^1 m^2 m^1 m^1$
428.			$\bar{1}_6^1 m^1 m^1$	478.	3m	222	$2z_6/2_6^2 z_6^{m_x m_y m_z}$
429.			$\bar{1}_6^2 m^1 m^1$	479.		mm2	$m_x 6/m_y m^1 m^1 m^1$
430.			$m_6^1 m^1 m^1$	480.			$m_x 6/2_6^2 m^1 m^1 m^1$
431.			$m_6^2 m^1 m^1$	481.			$2z_6/m_x m^1 m^1 m^1$
432.		m	$3z_6^2 z_6^{m_x m_y m_z}$	482.		2/m	$m_6/\bar{1}_1 m^1 m^1 m^1$
433.		3m	$3z_6^{m_x m_y m_z}$	483.			$\bar{1}_6/m^1 m^1 m^1 m^1$
434.	1	622	$6z_6^2 z_6^{m_x m_z}$	484.			$m_6/2_6^2 m^1 m^1 m^1$
435.		$\bar{3}m$	$\bar{3}z_6^{m_x m_y m_z}$	485.			$2_6/m^1 m^1 m^1 m^1$
436.			$\bar{3}z_6^2 z_6^{m_x m_y m_z}$	486.			$\bar{1}_6/2_6^2 m^1 m^1 m^1$
437.		6mm	$6z_6^{m_x m_z}$	487.			$2_6/\bar{1}_1 m^1 m^1 m^1$
438.		$\bar{6}m2$	$\bar{6}z_6^{m_x m_z}$	488.	32	222	$2z_6/2_6^2 m^2 y m^2 z_6^m$
439.			$\bar{6}z_6^2 z_6^{m_x m_z}$	489.		mm2	$m_y 6/2_6^2 z_6^{m_x m_z}$
440.	6/mmm	6/mmm	$1_6/1_6^1 m^1 m^1 m^1$	490.			$m_y 6/m_x m^2 z_6^{m_y m_z}$
441.		$\bar{3}m$	$2_6/2_6^1 m^1 m^1 m^1$	491.			$2z_6/m_x m_y m_z$
442.		m	$m_6/m^1 m^1 m^1 m^1$	492.		2/m	$m_6/2_6^2 m^1 m^1 m^1$
443.		$\bar{1}$	$\bar{1}_6/\bar{1}_1 m^1 m^1 m^1 m^1$	493.			$\bar{1}_6/2_6^2 m^1 m^1 m^1$
444.		$\bar{6}m2$	$2_6/1_6^2 m^1 m^1 m^1$	494.			$2_6/\bar{1}_1 m^2 m^1 m^1$
445.		2	$m_6/1_6^1 m^1 m^1 m^1$	495.			$2_6/\bar{1}_1 m^1 m^1 m^1$
446.		$\bar{1}$	$\bar{1}_6/1_6^1 m^1 m^1 m^1$	496.			$\bar{1}_6/m^1 m^1 m^1 m^1$
447.		6/m	$1_6/1_6^1 m^1 m^1 m^1$	497.			$2_6/m^1 m^1 m^1 m^1$
448.		m	$1_6/1_6^1 m^1 m^1 m^1$	498.		2/m	$3z_6/1_6^2 z_6^{m_x m_y m_z}$
449.		$\bar{1}$	$1_6/1_6^1 m^1 m^1 m^1$	499.		3m	$3z_6/1_6^1 m_x m_y m_z$
450.		6mm	$1_6/2_6^1 m^1 m^1 m^1$	500.		3	$m_x 6/2_6^2 z_6^{m_x m_y m_z}$

Table 1 (cont.)

R	r	B		R	r	B	
501.			$2z_6/mz_m^mxy_m$	551.	$\bar{4}3m$	1	$1_4^{-1}3^1_1m$
502.			$2z_6/mz_m^m2x_m2y_m$	552.		23	$2_4^{-1}3^2_m$
503.			$mz_6/m^m2x_m^m2y_m$	553.		m	$m_4^{-1}3^m_m$
504.			$mz_6/2x_m^m2y_m^m$	554.		1	$\bar{1}_4^{-1}3^1_1m$
505.			$2z_6/2x_m^m2y_m^m$	555.	222	32	$2x_4^3z_3^2x_m$
506.			$2z_6/m^m2x_m^m2y_m^m$	556.		3m	$m_4^{-3}z_3^m$
507.			$mz_6/2x_m^m2y_m^m$	557.	1	432	$4z_4^3xy_3^2xy_m$
508.			$mz_6/m^m2x_m^m2y_m^m$	558.		$\bar{4}3m$	$\bar{4}z_4^3xy_3^2xy_m$
509.			$mz_6/\bar{1}_m^m2x_m^m2y_m^m$	559.	432	432	$1_4^13^1_12$
510.			$2z_6/m^m2x_m^m2y_m^m$	560.		23	$2_4^13^2_2$
511.			$2z_6/2x_m^m2y_m^m$	561.		m	$m_4^13^m_2$
512.			$2z_6/\bar{1}_m^m2x_m^m2y_m^m$	562.		$\bar{1}$	$\bar{1}_4^13^1_12$
513.			$2z_6/\bar{1}_m^m2x_m^m2y_m^m$	563.	222	32	$2x_4^3z_3^2x_2$
514.			$\bar{1}_6/m^m2x_m^m2y_m^m$	564.		3m	$m_4^3z_3^m2x_2$
515.			$\bar{1}_6/2x_m^m2y_m^m$	565.	1	432	$4z_4^3xy_3^2xy_2$
516.	$\bar{1}$	622	$6z_6/2x_m^m2y_m^m$	566.		$\bar{4}3m$	$\bar{4}z_4^3xy_3^2xy_2$
517.		$\bar{3}m$	$\bar{3}z_6/\bar{1}_m^m2x_m^m2y_m^m$	567.	$4/m\bar{3}2/m$	$4/m\bar{3}2/m$	$1_4^1/m^13^1_12/1^1m$
518.		6mm	$6z_6/2x_m^m2y_m^m$	568.		$2/m\bar{3}$	$2_4^1/m^13^1_12/2^2m$
519.		$\bar{6}m2$	$\bar{6}z_6/m^m2x_m^m2y_m^m$	569.		m	$m_4^1/m^13^1_12/2^2m$
520.	m	622	$6z_6/1_m^m2x_m^m2y_m^m$	570.		$\bar{1}$	$\bar{1}_4^1/m^13^1_12/2^2m$
521.		$\bar{3}m$	$\bar{3}z_6/1_m^m2x_m^m2y_m^m$	571.		$\bar{4}3m$	$2_4^1/2^2z_3^2/1^1m$
522.		6mm	$6z_6/1_m^m2x_m^m2y_m^m$	572.		m	$m_4^1/m^m3^m2/1^1m$
523.		$\bar{6}m2$	$\bar{6}z_6/1_m^m2x_m^m2y_m^m$	573.		$\bar{1}$	$\bar{1}_4^1/m^m3^m2/1^1m$
524.	2	622	$3z_6/2x_m^m2y_m^m$	574.		432	$1_4^1/2^2m^23^1_12/2^2m$
525.		$\bar{3}m$	$3z_6/\bar{1}_m^m2x_m^m2y_m^m$	575.		m	$1_4^1/m^m3^m2/2^2m$
526.		6mm	$3z_6/\bar{1}_m^m2x_m^m2y_m^m$	576.		$\bar{1}$	$1_4^1/\bar{1}_m^m3^m2/2^2m$
527.		$\bar{6}m2$	$3z_6/m^m2x_m^m2y_m^m$	577.	23	222	$2x_4^1/2^2y_3^2z_3^2x_2/2^2z_m$
528.		6mm	$3z_6/m^m2x_m^m2y_m^m$	578.		mm2	$2z_4/m^m2x_m^m3^m2z_2/m^m$
529.		6mm	$3z_6/m^m2x_m^m2y_m^m$	579.		2/m	$m_4^1/2^2z_m^2z_3^m2/m^m$
530.	1	$6/mmm$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	580.		2/m	$m_4^1/m^m3^m2/2^2m$
531.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	581.		2/m	$2_4^1/\bar{1}_m^m3^m2/2^2m$
532.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	582.		$\bar{1}$	$2_4^1/m^m3^m2/2^2m$
533.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	583.		$\bar{1}$	$\bar{1}_4^1/2^2m^23^1_12/2^2m$
534.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	584.		$\bar{1}$	$\bar{1}_4^1/m^m3^m2/2^2m$
535.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	585.		$\bar{1}$	$m_4^1/2^2m^23^m2/2^2m$
536.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	586.		$\bar{1}$	$m_4^1/\bar{1}_m^m3^m2/2^2m$
537.		$6/m^m$	$6z_6/\bar{1}_m^m2x_m^m2y_m^m$	587.	mmm	32	$2x_4^1/m^3z_3^2x_2/2^2x_m$
538.	23	23	1_2^13	588.		3m	$m_4^1/m^3z_3^m2/m^m$
539.		222	1_2^33	589.	222	622	$2z_4/2^2z_m^m6z_3^2z_2/2^2x_m$
540.		1	$2z_2^3xy_3$	590.		$\bar{3}m$	$2x_4/1_m^m3^m2z_3^2x_2/m^m$
541.	$2/m\bar{3}$	$2/m\bar{3}$	$1_2/1_m^13$	591.		$\bar{3}m$	$m_4^1/1_m^m3^m2z_3^2x_2/2^2x_m$
542.		23	$1_2/2^2m^3$	592.		6mm	$m_4^1/2^2z_m^m6z_3^m2/m^m$
543.		m	$1_2/m^m3$	593.		$\bar{6}m2$	$m_4^1/m^m2z_m^m6z_3^m2/2^2x_m$
544.		$\bar{1}$	$1_2/\bar{1}_m^m3$	594.		$\bar{6}m2$	$2z_4/m^m2z_m^m6z_3^2z_2/m^m$
545.	mm	3	$1_2/1_m^33$	595.		$\bar{1}$	$4z_4/2^2z_m^m3xy_3^2xy_2/2^2xy_m$
546.	222	6	$1_2/2^2m^3$	596.		$\bar{4}3m$	$\bar{4}z_4/2^2z_m^m3xy_3^2xy_2/m^m$
547.		$\bar{3}$	$1_2/\bar{1}_m^33$	597.	1	$4/m\bar{3}2/m$	$4z_4/m^m3xy_3^2xy_2/m^m$
548.		$\bar{6}$	$1_2/m^63$	598.		$\bar{4}3m$	$\bar{4}z_4/m^m3xy_3^2xy_2/2^2xy_m$
549.	$\bar{1}$	23	$2z_2/2z_m^3xy_3$				
550.	1	$2/m\bar{3}$	$2z_2/m^m3xy_3$				

notation (*International Tables for X-ray Crystallography*, 1952). We have chosen the notation as such so as to have the most concise international notation which contains symbols representing a set of generators of the point group. In the fourth column of Table 1 we have listed a representative spin point group from each class of spin point groups \mathbf{R}_s , defined by isomorphisms between \mathbf{R}/\mathbf{r} and \mathbf{B} . The notation used to denote a spin point group \mathbf{R}_s , belonging to the family of \mathbf{B} and \mathbf{R} is a modified international notation of the point group \mathbf{R} . Each symbol in the international notation of a point group \mathbf{R} represents an element R of \mathbf{R} . This element of \mathbf{R} appears in a spin point group belonging to the family of \mathbf{B} and \mathbf{R} paired with one element of \mathbf{B} , i.e. in the spin point group element $[B|R]$ of \mathbf{R}_s . The notation used for the spin group \mathbf{R}_s is constructed from the international notation for the point group \mathbf{R} by replacing the symbol R for each element in the international notation for \mathbf{R} by the symbol ${}^B R$, where ${}^B R$ represents the element $[B|R]$ of \mathbf{R}_s . For example, consider the class of spin point groups #103 where $\mathbf{R}=4/m$, $\mathbf{r}=2/m$, and $\mathbf{B}=2$. The spin point group defined by the isomorphism between \mathbf{R}/\mathbf{r} and \mathbf{B} is:

$$\mathbf{R}_s = [1|2/m] + [2|4] [1|2/m].$$

Since $\mathbf{R}=4/m$ and $[2|4]$ and $[1|m]$ are elements of \mathbf{R}_s , this spin point group is denoted by ${}^{24}4/m$.

In a spin point group belonging to a family of \mathbf{B} and \mathbf{R} , the elements of \mathbf{R} are defined with respect to a coordinate system x, y, z in physical space, and elements of \mathbf{B} with respect to a coordinate system $\bar{x}, \bar{y}, \bar{z}$ in spin space. The two coordinate systems are arbitrarily mutually orientated. The positions of the symbols of the various rotations in the international notation of point groups imply the mutual orientation of the axes of rotation of the corresponding rotations. Consequently, in the modified international notation used for spin point groups, the symbols representing elements of \mathbf{R} do not need subindices to denote the orientation of the axes of rotation. However, to avoid ambiguities we have in many cases attached subindices to elements of the point group \mathbf{B} . For example, consider the spin point group #64 of Table 1 where $\mathbf{R}=mmm$, $\mathbf{r}=2$, and $\mathbf{B}=222$. The notation for this group with all subindices included is ${}^{2\bar{z}}m_x{}^{2\bar{z}}m_y{}^{2\bar{x}}m_z$ but following the 'international' conventions, the subindices on elements of \mathbf{R} can be left out, and it is listed as ${}^{2\bar{z}}m{}^{2\bar{z}}m{}^{2\bar{x}}m$ (In Table 1 for typographical simplicity, the bars on the spin space coordinates have been deleted). The remaining indices cannot be deleted without causing ambiguities, e.g. compare with spin point group #80, ${}^{2\bar{x}}m{}^{2\bar{y}}m{}^{2\bar{z}}m$. However, no subindices

are necessary in the spin point group #61, ${}^2m{}^2m{}^2m$, since in this case $\mathbf{B}=2$.*

Elements of magnetic point groups have been denoted by $[R, A(R)]$ where $A(R)$ is an element of the time-inversion group consisting of the identity E and time inversion E' (Litvin & Opechowski, 1974). In the notation used for spin point groups, an element of a magnetic group is written as $[\delta_{A(R)}\delta_R R||R]$ where $\delta_R = \det R$, $\delta_{A(R)} = +1$ if $A(R)=E$, and $\delta_{A(R)} = -1$ if $A(R)=E'$. Magnetic point groups are special cases of spin point groups, and in column five of Table 1 we have indicated, by giving the international notation for the corresponding magnetic point groups (Opechowski & Guccione, 1965), to which class of trivial point groups belongs each of the 32 classes of trivial magnetic point groups and 58 classes of non-trivial magnetic point groups.

The author would like to thank Dr S. Dembinski for translating into English the preprints by Koptsik & Kotsev (1974). This work was supported by a grant from the National Research Council of Canada.

* After this work was completed we have received a preprint of a Communication of the Joint Institute of Nuclear Research, Dubna (Koptsik & Kotsev, 1974a) where generalizations of point groups, mathematically similar to those discussed here, are described. However, their work deals with generalizations where the group denoted here by \mathbf{B} is a group of permutations. A referee of this paper has also pointed out the recent work by Harker (1976) where again, as in Koptsik & Kotsev (1974a), generalizations of point groups are described where the group \mathbf{B} is a group of permutations. We have also received, from a second referee, a second preprint of a Communication of the Joint Institute of Nuclear Research, Dubna (Koptsik & Kotsev, 1974b) where groups identical to the groups tabulated in this paper have been derived.

References

- HARKER, D. (1976). *Acta Cryst.* A32, 133–139, (erratum: A32, 675).
International Tables for X-ray Crystallography (1952). Vol. I. Birmingham: Kynoch Press.
 KOPTSIK, N. A. & KOTSEV, J. N. (1974a). Communication of the Joint Institute for Nuclear Research, Dubna, USSR (P4-8067).
 KOPTSIK, N. A. & KOTSEV, J. N. (1974b). Communication of the Joint Institute for Nuclear Research, Dubna, USSR (P4-8466).
 LITVIN, D. B. (1971). Ph. D. Thesis, Technion-Israel Institute of Technology.
 LITVIN, D. B. (1973). *Acta Cryst.* A29, 651–660.
 LITVIN, D. B. (1974). *Acta Cryst.* A30, 785–791.
 LITVIN, D. B. & OPECHOWSKI, W. (1974). *Physica*, 76, 538–554.
 OPECHOWSKI, W. & GUCCIONE, R. (1965). *Magnetism*, Vol. IIA, pp. 105–165, Edited by G. T. RADO & H. SUHL. New York: Academic Press.